Abstract

A subset $S \subseteq V$ in a graph $G = (V, E)$ is an $[j, k]$-set, if for every vertex $v \in V \setminus S$, $j \leq |N(v) \cap S| \leq k$ for non-negative integers $j$ and $k$, that is, every vertex $v \in V \setminus S$ is adjacent to at least $j$ but not more than $k$ vertices in $S$. In this paper, we focus on small values of $j$ and $k$, and relate the concept of $[j, k]$-sets to a host of other concepts in domination theory, including perfect domination, efficient domination, nearly perfect sets, 2-packings, and $k$-dependent sets. We also determine bounds on the cardinality of minimum $[1, 2]$-sets and investigate extremal graphs achieving these bounds. This study has implications for restrained domination as well. Using a result for $[1, 3]$-sets, we show that for any grid graph $G$, the restrained domination number is equal to the domination number of $G$. 