A Contribution to Upper Domination, Irredundance and Distance-2 Domination in Graphs

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Abstract

Let \( G = (V, E) \) be a graph. The open neighborhood of a vertex \( v \in V \) is the set \( N(v) = \{u | uv \in E \} \) and the closed neighborhood of \( v \) is the set \( N[v] = N(v) \cup \{v\} \). The open neighborhood of set \( S \) of vertices is the set \( N(S) = \bigcup_{v \in S} N(v) \), while the closed neighborhood of a set \( S \) is the set \( N[S] = \bigcup_{v \in S} N[v] \). A set \( S \subseteq V \) dominates a set \( T \subseteq V \) if \( T \subseteq N[S] \), written \( S \rightarrow T \). A set \( S \subseteq V \) is a dominating set if \( N[S] = V \); and is a minimal dominating set if it is a dominating set, but no proper subset of \( S \) is also a dominating set; and is a \( \gamma \)-set if it is a dominating set of minimum cardinality. In this paper we consider the family \( \mathcal{D} \) of all dominating sets of a graph \( G \), the family \( \mathcal{MD} \) of all minimal dominating sets of a graph \( G \), and the family \( \gamma \) of all \( \gamma \)-sets of a graph \( G \). The study of these three families of sets provides new characterizations of the distance-2 domination number, the upper domination number and the upper irredundance number in graphs.